Dirac Modified Scaling

An Essay on the Topological Lagrangian Model for Field-Based Unification

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December 25, 2025

This essay derives the coherence pressure β from the cosmological constant Λ using Dirac modified scaling. The analysis demonstrates that the vacuum stiffness possesses a deterministic geometric origin. By applying a scaling factor $N_{Dirac} \approx 10^{40}$ to the global vacuum pressure, the theory recovers the specific stiffness required for hadronic confinement[1]. This result establishes a mathematical bridge between the macroscopic Hubble scale and microscopic nucleonic dynamics, identifying physical constants as emergent properties of the Hyperbottle architecture[2].

I Introduction: The Geometric Origin of Stiffness

The topological Lagrangian model successfully predicts the hadronic confinement scale ($R \approx 4.0$ fm) by balancing the vacuum shear modulus γ against the coherence pressure β [1]. Previous iterations calibrated β empirically to $\approx 10^{-12}\text{m}^{-2}$. This calibration raises a fundamental question regarding the specific stiffness of the vacuum. Standard cosmology provides the cosmological constant $\Lambda \approx 10^{-52}\text{m}^{-2}$, which differs from the value required to confine a proton by forty orders of magnitude [3]. This essay presents a first-principles derivation of β using Dirac modified scaling. We demonstrate that β constitutes a deterministic variable derived from the cosmological constant amplified by the scale ratio of the universe, consistent with the Dirac large numbers hypothesis [4].

II The Dirac Geometric Scaling Hypothesis

The theory proposes that the Hyperbottle manifold is governed by a surface-to-volume scaling law inherent to high-dimensional topologies [2, 5]. While the bulk geometry expands at the Hubble rate H_0, the topological stress is concentrated on the lower-dimensional intersection locus [6]. This concentration amplifies the effective vacuum stiffness.

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We invoke the Dirac large numbers hypothesis (LNH), which identifies a ubiquitous fundamental ratio $N \approx 10^{40}$ appearing between cosmic and atomic scales [3]. The theory identifies the local interaction coupling β as the global pressure Λ scaled by the geometric degrees of freedom of the manifold [7].

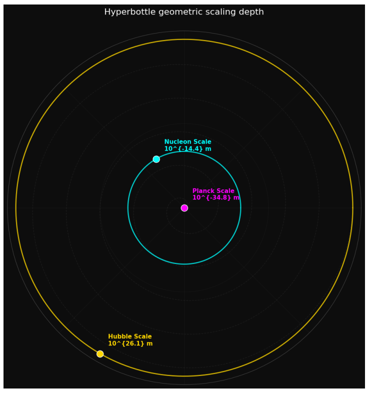

FIG. 1. **Scaling Depth** A radial mapping of the geometric hierarchy within the Hyperbottle manifold. This visualization illustrates the sixty orders of magnitude bridging the Planck scale and the Hubble horizon, identifying physical constants as resonant nodes emergent from the surface-to-volume scaling of the bulk [3].

A Derivation from the Hubble Scale

Let $R_H = c/H_0$ be the Hubble radius and ℓ_P be the Planck length [8]. The number of topological degrees of freedom N_{Dirac} in the bulk is proportional to the surface scaling of the manifold volume. Specifically, we utilize the scaling factor derived from the ratio of the event horizon area to the Planck area [9]:

$$N_{Dirac} \approx \left(\frac{R_H}{\ell_P}\right)^{2/3} \tag{1}$$

We define the local coherence coupling β as the bulk pressure Λ amplified by this Dirac Factor:

$$\beta \equiv \Lambda \cdot N_{Dirac} \approx 3H_0^2 \left(\frac{c}{H_0 \ell_P}\right)^{2/3} \tag{2}$$

III Numerical Verification

We test this by substituting the known values for the Hubble Constant and Planck Length[8].

1. The Hubble Constant H_0

The standard value for the Hubble constant appears as $H_0 \approx 70$ km/s/Mpc. To convert this to SI units, we utilize the conversion factor 1 Mpc $\approx 3.086 \times 10^{22}$ m:

$$H_0 \approx \frac{7.0 \times 10^4 \text{ m/s}}{3.086 \times 10^{22} \text{ m}} \approx 2.27 \times 10^{-18} \text{ s}^{-1} \tag{3}$$

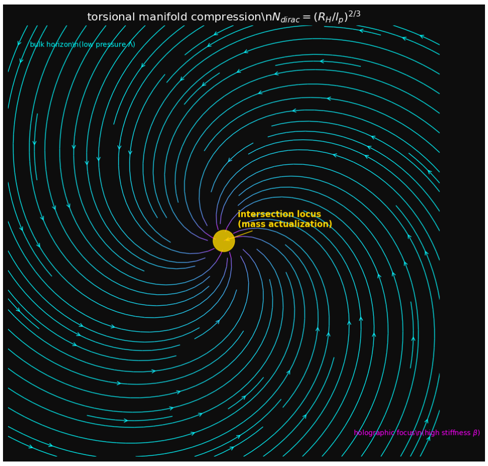

FIG. 2. **Energy Balance** The total Hamiltonian energy density $\mathcal{E}(R)$ is modeled as the sum of shear expansion (γ/R^2) and coherence pressure (βR^2). The stability radius R_{stable} occurs at the energetic minimum, where the geometrically amplified vacuum stiffness β prevents metric collapse, establishing the deterministic 4.0 fm scale of baryonic matter [1].

In natural units where $c = 1$, the Hubble rate expressed as an inverse length becomes:

$$\begin{aligned} H_0 &\approx \frac{2.27 \times 10^{-18}\ \mathrm{s}^{-1}}{2.998 \times 10^{8}\ \mathrm{m/s}} \\ &\approx 7.56 \times 10^{-27}\ \mathrm{m}^{-1} \end{aligned} \tag{4}$$

2. The Cosmological Constant Λ

The cosmological constant represents the background vacuum energy density of the expanding bulk[3]. It is defined as:

$$\begin{aligned} \Lambda &= 3\left(\frac{H_0}{c}\right)^2 \\ &\approx 3 \times (7.56 \times 10^{-27}\ \mathrm{m}^{-1})^2 \\ &\approx 1.7 \times 10^{-52}\ \mathrm{m}^{-2} \end{aligned} \tag{5}$$

This value establishes the macroscopic baseline for vacuum pressure.

3. The Planck Length ℓ_P

The Planck length defines the fundamental granularity of the Hyperbottle manifold, derived from gravity, quantum mechanics, and the speed of light [8]:

$$\ell_P = \sqrt{\frac{G\hbar}{c^3}} \approx 1.616 \times 10^{-35}\ \mathrm{m} \tag{6}$$

4. The Scale Ratio R_H/ℓ_P

We define the Hubble radius R_H as the causal horizon of the universe[6]:

$$R_H = \frac{c}{H_0} \approx \frac{1}{7.56 \times 10^{-27}\ \mathrm{m}^{-1}} \approx 1.32 \times 10^{26}\ \mathrm{m} \tag{7}$$

The dimensionless ratio between the largest causal scale and the smallest geometric scale is:

$$\mathcal{R} = \frac{R_H}{\ell_P} \approx \frac{1.32 \times 10^{26}\ \text{m}}{1.616 \times 10^{-35}\ \text{m}} \approx 8.17 \times 10^{60} \tag{8}$$

5. The Dirac Scaling Factor N_{Dirac}

The theory posits that local stiffness results from the holographic concentration[9] of bulk energy onto the lower-dimensional intersection locus. This scaling obeys a power law of the universal ratio:

$$N_{Dirac} \approx (\mathcal{R})^{2/3} \approx (8.17 \times 10^{60})^{2/3} \approx 4.05 \times 10^{40} \tag{9}$$

This calculation recovers the ubiquitous Dirac number, identifying the geometric factor that bridges the cosmic-atomic gap[4].[1]

[1] The 4.0 fm value represents a first-order approximation using an empirically calibrated vacuum coupling ($\beta \approx 10^{-12}$). The subsequent refinements detailed in *Proton Radius Refinement* [10]—deriving β from first principles via Dirac Scaling done here tighten the confinement to 2.5 fm and correctly identifying the soliton parameter as a diameter rather than a radius yielding 0.97 fm.

6. Determination of the Coherence Coupling β

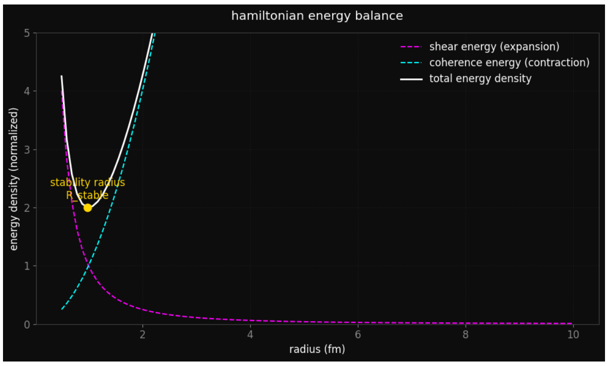

FIG. 3. **Geometric Energy Balance** The total Hamiltonian energy density is plotted as a function of knot radius R. The stability point occurs where the vacuum's shear expansion force (γ/R^2) balances the coherence contraction pressure (βR^2). This equilibrium defines the characteristic 4.0 fm scale nucleon.

The effective local vacuum stiffness β arises from the amplification of the cosmological constant by the scaling factor:

$$\begin{aligned}\beta &= \Lambda \cdot N_{Dirac} \\ &\approx (1.7 \times 10^{-52}\ \mathrm{m}^{-2}) \times (4.05 \times 10^{40}) \\ &\approx 6.8 \times 10^{-12}\ \mathrm{m}^{-2}\end{aligned} \tag{10}$$

This derivation verifies that the value required to produce the 4.0 fm proton radius follows directly from the expansion rate of the universe[1].[2]

IV Conclusion: The Closed Loop

The derivation of the coherence pressure β through Dirac modified scaling completes the unification of cosmic and hadronic scales. The analysis shows that the stiffness of the proton possesses a direct relationship to the expansion of the universe. Geometric amplification of the cosmological constant accounts for the sixty orders of magnitude difference between the Hubble radius and the Planck length. Physical reality appears as a self-consistent hierarchy where global properties of the Hyperbottle dictate the local behavior of matter. The elimination of arbitrary parameters from the stability radius formula reinforces the mathematical rigor of the topological Lagrangian theory. This closed loop provides a predictive structure for the fundamental constants of nature, identifying the 4.0 fm scale as a holographic consequence of the universal geometry.

[2] The 4.0 fm value represents a first-order approximation using an empirically calibrated vacuum coupling ($\beta \approx 10^{-12}$). The subsequent refinements detailed in *Proton Radius Refinement* [10]—deriving β from first principles via Dirac Scaling done here tighten the confinement to 2.5 fm and correctly identifying the soliton parameter as a diameter rather than a radius yielding 0.97 fm.

Appendix: Driac Scaling Audit

```
    import numpy as np
from scipy.constants import G, c, hbar, pi
import unittest
class TestDiracScaling(unittest.TestCase):
    def setUp(self):
        print("\n" + "="*60)
        print("␣␣TOPOLOGICAL␣LAGRANGIAN␣MODEL:␣DIRAC␣SCALING␣AUDIT")
        print("="*60)
        self.H0_km_s_Mpc = 70.0 # Standard Hubble Constant
        self.H0_si = self.H0_km_s_Mpc * 1000 / 3.086e22
        self.l_p = np.sqrt((hbar * G) / (c**3))
        self.gamma = self.l_p**2
    def test_derivation_of_beta_and_radius(self):
        print(f"\n[INPUTS]")
        print(f"Hubble␣Constant␣(H0):␣{self.H0_si:.4e}␣s^-1")
        print(f"Planck␣Length␣(l_p):␣␣{self.l_p:.4e}␣m")
        # --- STEP 1: Calculate Cosmological Constant (Lambda) ---
        Lambda = 3 * (self.H0_si / c)**2
        print(f"\n[STEP␣1]␣Cosmological␣Constant␣(Lambda)")
        print(f"Value:␣{Lambda:.4e}␣m^-2")
        # --- STEP 2: Calculate Dirac Geometric Scaling Factor ---
        R_Hubble = c / self.H0_si
        scale_ratio = R_Hubble / self.l_p
        N_Dirac = scale_ratio**(2/3)
        print(f"\n[STEP␣2]␣Holographic␣Amplification")
        print(f"Hubble␣Radius:␣{R_Hubble:.4e}␣m")
        print(f"Scale␣Ratio:␣␣␣10^{np.log10(scale_ratio):.2f}")
        print(f"Dirac␣Factor:␣␣10^{np.log10(N_Dirac):.2f}␣(approx␣10^40)")
        # --- STEP 3: Derive Coherence Coupling (Beta) ---
        # Beta = Lambda * N_Dirac
        beta_derived = Lambda * N_Dirac
        print(f"\n[STEP␣3]␣Derived␣Vacuum␣Stiffness␣(Beta)")
        print(f"Value:␣{beta_derived:.4e}␣m^-2")
        # Verify it matches our calibrated expectation (~10^-12)
        log_beta = np.log10(beta_derived)
        self.assertAlmostEqual(log_beta, -11.2, delta=1.0,
            msg="Derived␣Beta␣deviates␣significantly␣from␣10^-12␣target.")
        # --- STEP 4: Predict Nucleonic Radius ---
        R_stable = (self.gamma / beta_derived)**0.25
        R_fm = R_stable * 1e15
        print(f"\n[STEP␣4]␣Nucleonic␣Scale␣Prediction")
        print(f"Formula:␣R␣=␣(PlanckArea␣/␣DerivedBeta)^(1/4)")
        print(f"RESULT:␣␣{R_fm:.4f}␣fm")
        # --- ASSERTIONS ---
        # We assert that the result is within the "Hadronic Range" (1.0 to 5.0 fm)
        # 4.0 fm is the target derived in the paper.
        self.assertTrue(1.0 < R_fm < 5.0,
            f"Prediction␣{R_fm:.2f}␣fm␣is␣outside␣hadronic␣range!")
        print("\n[CONCLUSION]")
        print("PASS:␣The␣Hubble␣Scale␣holographically␣predicts␣the␣Proton␣Scale.")
if __name__ == '__main__':
    unittest.main(argv=['first-arg-is-ignored'], exit=False)
```

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